USN


15MAT31

Third Semester B.E. Degree Examination, Aug./Sept. 2020 Engineering Mathematics - III

Time: 3 hrs.
Max. Marks: 80
Note: Answer any FIVE full questions, choosing ONE full question from each module.

## Module-1

1 a. Obtain the Fourier series of $f(x)=x(2 \pi-x)$ in $0 \leq x \leq 2 \pi$ and hence deduce that :

$$
\begin{equation*}
\frac{\pi^{2}}{8}=\frac{1}{1^{2}}+\frac{1}{3^{2}}+\frac{1}{5^{2}}+ \tag{08Marks}
\end{equation*}
$$

b. Express y as a Fourier series upto the second harmonics given :

| x | 0 | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| y | 4 | 8 | 15 | 7 | 6 | 2 |

(08 Marks)

2 a. Obtain the Fourier series for $f(x)=e^{-x}$ in the interval $0<x<2$.
(06 Marks)
b. Expand the function $f(x)=x \sin x$ as a Fourier series in the interval $-\pi \leq x \leq \pi$.
(05 Marks)
c. Expand $f(x)=2 x-1$ as a cosine half range Fourier series in $0 \leq x<1$.

## Module-2

3 a. Find the Fourier transform of

$$
f(x)=\left\{\begin{array}{ccc}
1-|x| & \text { for } & |x| \leq 1 \\
0 & \text { for } & |x|>1
\end{array}\right.
$$

$$
\text { And hence deduce that } \int_{0}^{10} \frac{\sin ^{2} t}{\mathrm{t}^{2}} \mathrm{dt}=\frac{\pi}{2}
$$

(06 Marks)
b. Find the Fourier cosine transform of

$$
f(x)=\left\{\begin{array}{lc}
x & 0<x<2  \tag{05Marks}\\
0 & \text { else where }
\end{array}\right.
$$

c. Find the z - transform of :
i) $\cos n \theta$
ii) $\sin n \theta$.
(05 Marks)

## OR

4 a. Obtain the Fourier transform of $f(x)=x e^{-|x|}$.
(06 Marks)
b. If $u(z)=\frac{2 z^{2}+3 z}{(z+2)(z-4)}$, find the inverse $z$-transform.
(05 Marks)
c. Solve the difference equation $y_{n+2}+6 y_{n+1}+9 y_{n}=2^{n}$ with $y_{0}=y_{1}=0$ using $z-$ transforms.

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## Module-3

5 a. Compute the co-efficient of correlation and equation of lines of regression for the data :

| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| y | 9 | 8 | 10 | d 2 | 11 | 13 | 14 |

b. Fit a best fitting parabola $y=a x^{2}+b x+c$ for the following data :

| x | I | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| y | 10 | 12 | 13 | 16 | 19 |

(05 Marks)
c. Use the Regula - Falsi method to find a real root of the equation $x^{3}-2 x-5=0$ correct to three decimal places.
(05 Marks)

## OR

6 a. Find the co-efficient of correlation for the following data :

| x | 10 | 14 | 18 | 22 | 26 | 30 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| y | 18 | 12 | 24 | 6 | 30 | 36 |

(06 Marks)
b. Fit a least square geometric curve $\mathrm{y}=\mathrm{a}^{\mathrm{bx}}$ for the following data :

| $x$ | 0 | 2 | 4 |
| :---: | :---: | :---: | :---: |
| $y$ | 8.12 | 10 | 31.82 |

(05 Marks)
c. Use Newton - Raphson method to find a real root of the equation : $\log _{10}^{\mathrm{x}}=1.2$ correct to four decimal places that is near to 2.5 .
(05 Marks)

## Module-4

7 a. From the following table find the number of students who have obtained :
i) Less than 45 marks
ii) Between 40 and 45 marks.

| Marks | $30-40$ | $40-50$ | $50-60$ | $60-70$ | $70-80$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Number of students | 31 | 42 | 51 | 35 | 31 |

(06 Marks)
b. Find the Legrange's interpolation polynomial for the following values $y(1)=3, y(3)=9$,

$$
y(4)=30 \text { and } y(6)=132 .
$$

c. Evaluate $\int_{0}^{1} \frac{d x}{1+x}$ taking seven ordinates by applying Simpson's $\frac{3}{8}^{\text {th }}$ rule.

8 a. Give $\mathrm{u}_{20}=24.37, \mathrm{u}_{22}=49.28, \mathrm{u}_{29}=162.86$ and $\mathrm{u}_{32}=240.5$ find $\mathrm{u}_{28}$ by Newton's divided difference formula.
(06 Marks)
b. Extrapolate for 25.4 given the data using Newton's backward formula :

| x | 19 | 20 | 21 | 22 | 23 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| y | 91 | 100.25 | 110 | 120.25 | 131 |

(05 Marks)
c. Evaluate : $\int_{0}^{1} \frac{x}{1+\mathrm{x}^{2}} \mathrm{dx}$ by Weddle's rule taking seven ordinates.
(05 Marks)

## Module-5

9 a. Verify Green's theorem for $\oint_{C}\left(x y+y^{2}\right) d x+x^{2} d y$ where $C$ is the closed curve of the region bounded by $\mathrm{y}=\mathrm{x}$ and $\mathrm{y}=\mathrm{x}^{2}$.
(06 Marks)
b. Derive Euler's equation in the form $\frac{\partial f}{\partial y}-\frac{d}{d x}\left(\frac{\partial f}{\partial y^{1}}\right)=0$.

c. If $\overrightarrow{\mathrm{F}}=\mathrm{xyi}+\mathrm{yzj}+\mathrm{zxk}$ evaluate $\int_{\mathrm{C}} \overrightarrow{\mathrm{F}} \cdot \mathrm{d} \overrightarrow{\mathrm{r}}$ where C is the curve represented by $\mathrm{x}=\mathrm{t}, \mathrm{y}=\mathrm{t}^{2}$, $\mathrm{z}=\mathrm{t}^{3},-1 \leq \mathrm{t} \leq 1$.
(05 Marks)

## OR

10 a. Verify Green's theorem in the plane for $\int_{C}\left(x^{2}+y^{2}\right) d x+3 x^{2} y d y$ where $C$ is the circle $x^{2}+y^{2}=4$ traced in the positive sence.
(06 Marks)
b. Evaluate $\int_{C}\left(x y d x+x y^{2} d y\right)$ by Stoke's theorem $C$ is the square in the $x-y$ plane with the vertices $(1,0),(-1,0),(0,1)$ and $(0,1)$.
(05 Marks)
c. Prove that the geodesics on a plane are straight lines.

